Fund of Hedge Funds Portfolio Optimization Using the Omega Ratio

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It is interesting to ponder how new and radical theories and equations in financial analysis become widely accepted. It most likely is a function of simplicity, elegance of design, and ease of application. But it is even more interesting to observe how, once widely accepted, such concepts can be misapplied. Perhaps two of the most common examples are the application of the Sharpe ratio and mean variance optimization (MVO) to hedge fund portfolio analysis. To be sure, both of these equations are elegant and beautiful in design and both have changed the way the world analyzes investment options, but an overwhelming body of evidence suggests that they are not applicable to the asymmetrical nature of hedge fund returns. They don't apply because they don't accurately account for the impacts of the third and fourth moments of hedge fund return distributions, which are used to calculate skewness and kurtosis, respectively.

That said, it is nearly impossible to review performance information on a hedge fund without seeing the fund's Sharpe ratio on prominent display (assuming it is a compelling number). Likewise the most popular hedge fund databases and analytical programs all offer some type of MVO algorithm.

Fortunately another elegant measure, the Omega ratio, is attaining widespread recognition and acceptance. Keating and Shadwick (2002) clearly demonstrate how Omega captures these higher moments. It is an equation that adds to mean and variance, captures all of the higher moment information in the return distribution, incorporates sensitivity to return levels, and is intuitive and relatively easy to calculate.

The body of literature about constructing and optimizing portfolios of hedge funds continues to grow and the alternatives are varied. In most cases a comparison is made to MVO, which suffers from similar drawbacks as Sharpe when applied to hedge fund returns. DeSouza and Gokcan (2004) defined an approach that is modeled on asset allocation methodologies familiar in traditional investment management but takes into account the idiosyncrasies of hedge fund investing as an asset class. Amenc and Martellini (2002) took the approach of evaluating out-of-sample performance of an improved estimator of the covariance structure of hedge fund index returns, focusing on its use for optimal portfolio selection.

Objective

The purpose of this paper is to build upon Omega's intuitive nature and discuss and review how Omega can be used to optimize a fund of hedge funds portfolio. It is worth noting now, and we will expand on this later, that optimization through Omega is purely a quantitative pursuit and, as investment practitioners, we believe that while it is an important part of multimanager hedge fund portfolio construction, it is not a final solution. And we can find solace in this because no quantitative measure should be an end-all solution.

Our objective here is to present a method of "optimizing" a portfolio of hedge funds using the Omega ratio. For the purposes of our discussion we will focus on a portfolio of alternative investment managers with nonnormal returns but, of course, the underlying investments can be anything. We put optimizing in quotes because our analysis is not the sole input in determining the optimized portfolio.

Understanding the Omega Ratio

We believe that using the Omega ratio for this purpose is a rather intuitive decision based on the abilities and strengths of the measure. Those in agreement with the evidence that more traditional measures such as the Sharpe ratio and other MVO-type measures are inappropriate for alternative investments should appreciate Omega's lack of assumptions about distributions.

In fact, it is from this lack of assumptions that Omega derives its power. Because the measure takes into account the exact distribution of the returns in question, it inherently includes all of the information encoded in the moments: mean, variance, skewness, and kurtosis. Admittedly, Omega is backward-looking and thus fallible, but it does not have the added weakness of assuming normality. In addition, as Cascon and Shadwick (2006) put forth, if we compare investments that come from different affine families of distributions, we then cannot use standard deviation as a "common unit of risk," which also leads to a weakening of the arguments put forth by the Sharpe ratio and MVO, at least in the case of alternative investment manager analysis.

The Omega ratio is defined by Keating and Shadwick (2002) to be:

$$\Omega(r) = \frac{\int_r^b (1 - F(x))dx}{\int_a^r F(x)dx}$$

where [*a,b*] is the interval of returns with a cumulative distribution function *F(x),* and *r* is the threshold return where the ratio is being evaluated. Clearly, a higher Omega ratio is better than a lower one, because it indicates either a greater probability-weighted gain or a smaller probability-weighted loss, or both, based on historical returns.

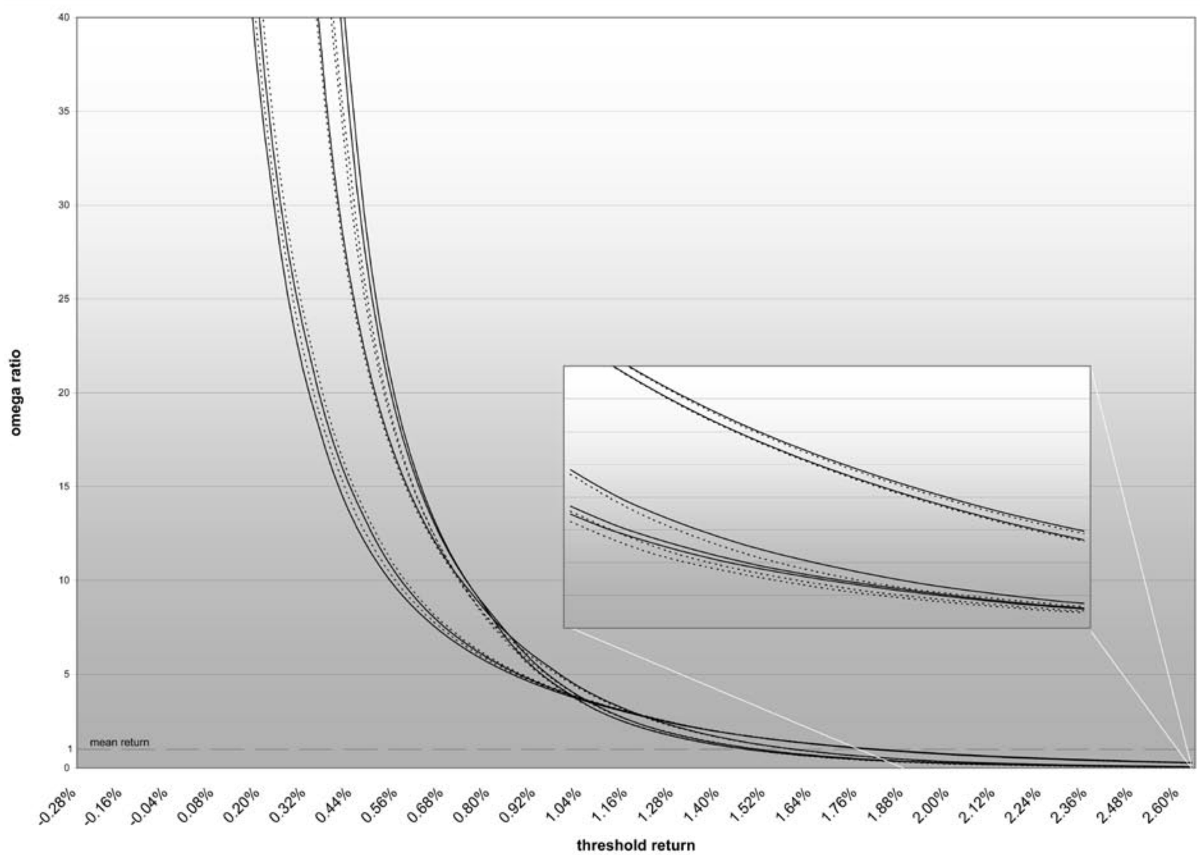
Omega as an Optimizer

As posited by Favre-Bulle and Pache (2003), a portfolio can be optimized using the Omega ratio at a specified threshold which, depending on the investor, can be selected in a number of ways. In this case, the threshold can be a minimum acceptable return, which can be anything from zero to the risk-free rate to virtually anything else. Optimizing at a given threshold is rather simple and not unlike performing a traditional MVO. Instead of minimizing variance while maximizing return, one minimizes probability-weighted

TABLE 1 Ordinary Risk Statistics

	ANNUALIZED STANDARD DEVIATION	ANNUALIZED RATE OF RETURN
Fund A	2.80%	14.60%
Fund B	20.97%	49.65%
Fund C	6.66%	33.42%
Fund D	6.56%	13.88%
Fund E	1.51%	16.11%
Fund F	6.45%	9.55%
Fund G	1.69%	7.42%
Fund H	3.94%	16.57%
Fund I	0.38%	10.50%
Fund J	2.80%	18.48%
Fund K	8.74%	19.57%

FIGURE 1 Comparison of Omega Plots



>> "PORTFOLIO OPTIMIZATION" CONTINUED

loss and maximizes the probability-weighted gain as calculated using the Omega ratio.

Our goal is to apply this procedure in a similar fashion using the Omega ratio but across an entire range of threshold returns, thereby resulting in what we believe is a rather robust optimization of sorts. We say "of sorts" because in performing this procedure across an entire range of thresholds it becomes somewhat challenging to define what a best portfolio would be, and thus we need to address other quantitative issues related to market factors impacting particular strategies and more qualitative issues related to specific investor needs and goals to ultimately come up with a "best" portfolio.

And so we can backtrack a bit now to examine in more detail the quantitative aspect of choosing the best portfolio. Favre-Bulle and Pache (2003) did this at individual thresholds by plotting efficient frontiers of probability-weighted losses versus probability-weighted gains. We extend this by calculating entire Omega ratio curves for large numbers of portfolios. As one can imagine, the mechanics of the calculations are quite straightforward while the challenge lies in actually defining and parsing out the best portfolios, a process that can be loosened or tightened to produce more or fewer results, respectively.

This netting out of best portfolios can be rather easy for a subjective human eye (albeit it can get rather difficult when there are thousands, if not millions, of results), but it can be a surprisingly quirky problem for objective and, well, inhuman computer code.

Suffice it to say, our method examined several aspects of each portfolio's Omega profile, including the thresholds where Omega existed, the spread or steepness of the plot in different directions, which corresponds to dispersion around the mean (i.e., variance) to the upside and downside, as well as the value of the Omega ratio itself in comparison to others across the entire range of thresholds.

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Analysis

We will argue that the number of funds being optimized is not material to the study, provided the returns streams reflect nonnormality. Thus, we optimized a portfolio of 11 underlying single manager hedge funds with the ordinary risk statistics shown in table 1.

The optimization was run with very loose restrictions with regard to minimums and maximums per investment, a liberty that would not normally be taken in a real-world application, but it nonetheless is one that allows us to produce meaningful results for illustrative purposes.

Figure 1 shows an Omega plot of our optimizer results (solid lines) along with the Omega plots of the portfolios suggested by a typical MVO (dashed lines).

Conclusion

Based on our understanding of Omega, which in the smallest of nutshells states that a higher value is better, it becomes evident that our optimization here has indeed produced a number of noteworthy portfolios compared with those produced by an MVO. We can see that the Omega optimized portfolios have better profiles on parts of the interval but become less desirable at other portions. Here, however, is where a somewhat more subjective decision would be made based on any number of factors that could include such things as investor risk aversion and investor goals. M

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